# Singular Value Decomposition (SVD) in Image Processing

## Applications, Techniques, and Findings

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# Objective, Methodology and Approach

- Objective: To explore Singular Value Decomposition (SVD) applications in image processing.
- Methodology :
  - Replication of Sadek's Works :
    - Reproduce key findings and techniques from Sadek's research on SVD in image processing.
    - Conduct mathematical experiments to evaluate and validate SVD-based image compression, denoising, and watermarking techniques.

# Introduction

- **Singular Value Decomposition (SVD)** is a powerful matrix factorization tool widely used in image processing.
- **Applications** include:
  - Image Compression
  - Image Denoising
  - Digital Watermarking for forensics
- **Objective**: Validate SVD's effectiveness and explore enhancements across these applications.

# SVD Fundamentals

- SVD decomposes a matrix $X$ into:

$$X = U \Sigma V^T$$

- Properties:
  - **Rank Approximation**: Reduces dimensionality by focusing on dominant singular values.
  - **Energy Compaction**: Most image energy is captured in the largest singular values, allowing effective compression [2, 3].

# Image Compression using SVD

- **Goal**: Minimize storage while preserving key image details.
- **Method**: Retain only top-$k$ singular values to approximate the original image.

## SVD reconstruction formula

$$X \approx X_{k=40} = \sum_{i=1}^{40} \sigma_i \cdot u_i \cdot v_i^T$$

- where:
  - $\sigma_i$ is the $i$-th singular value,
  - $u_i$ is the $i$-th left singular vector (column of $U$),
  - $v_i$ is the $i$-th right singular vector (column of $V$).

# Visual Comparison


**Figure:** Reconstructed image using SVD with low-rank approximation (k=40).

# Compression Method Comparison

| Method | MSE | PSNR (dB) | SSIM |
|---|---|---|---|
| SVD | 36.18 | 32.55 | 0.8247 |
| Discrete Cosine Transform | 107.66 | 27.81 | 0.8217 |
| Wavelet Transform (Haar) | 32.94 | 32.95 | 0.9582 |
| Fractal Compression | 20.47 | 35.02 | 0.9320 |
| Run-Length Encoding | 0.00 | $\infty$ | 1.000 |
| Predictive Coding | 107.25 | 27.83 | 0.5477 |

**Table:** Comparison of Image Compression Methods

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# Correlation between the truncation factor and PSNR and SSIM metrics

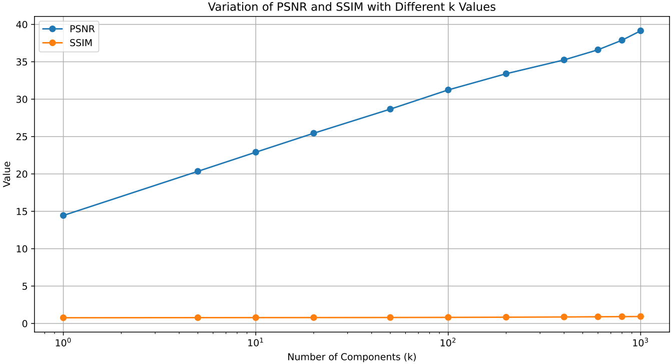


**Figure:** Variation of PSNR and SSIM with respect to the truncation factor $k$.

# Image Denoising with SVD

- **Goal**: Suppress noise without significant loss in image content.
- **Method**: Filter smaller singular values that represent noise.
- **Dynamic Thresholding**: $0.618 \times \text{mean}(S)$.
- **Results**:
  - PSNR Improvement: 12.42 (noisy) to 20.31 (denoised)
  - SSIM Improvement: 0.0324 (noisy) to 0.4374 (denoised)

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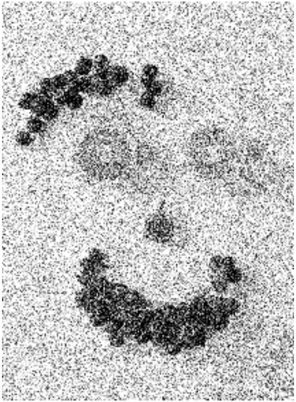
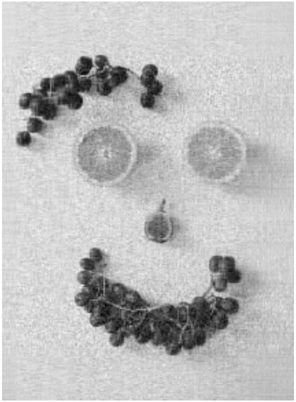
# Visual Comparison

**(a)** Original Image in JPEG format.



**(b)** Noisy Image (PSNR: 12.42, SSIM: 0.0324.)



**(c)** Denoised (PSNR:20.31, SSIM: 0.437.)

**Figure:** Comparison of Original, Noisy, and Denoised Images using SVD.

# SVD Denoising on BSD400 Dataset

(a) Original Image from the BSD400 dataset.

(b) Noisy Image (PSNR: 30.27, SSIM: 0.7794).

(c) Denoised (PSNR: 32.27, SSIM: 0.8636).

**Figure:** Comparison of Original, Noisy, and Denoised images using SVD on BSD400 sample image.

# Image Forensics - Watermarking with SVD

- **Purpose**: Embed secure watermarks for authenticity verification.
- **Techniques**:
  - Scaled Additive Approach: Adds scaled watermark data to singular values [1].
    $$SV_{\text{modified}} = SV_{\text{original}} + \alpha \cdot \text{Watermark}$$
  - Adaptive Scaled Additive (ASA): Fine-tunes watermark strength for resilience.
- **Formula**:

$$\text{SV}_{\text{mod}} = (1 - \alpha) \cdot \text{SV}_{\text{img}} + \alpha \cdot \text{Watermark}$$

# Image Forensic Workflow

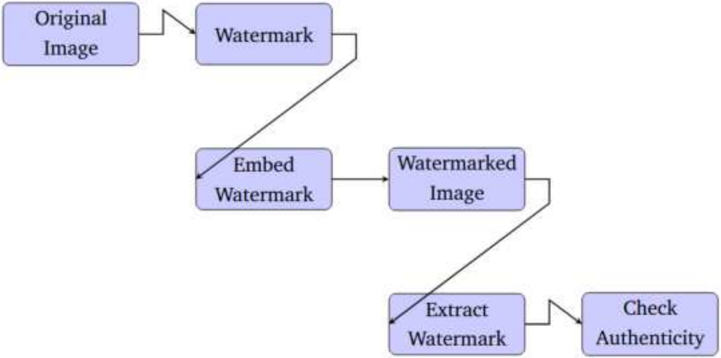



**Figure:** General image forensic workflow.

# Watermarking Results and Comparison

**Table:** Peak Signal to Noise Ratio of various watermarked versions of `test_077` image from BSD400 dataset under scaled additive (SA) and adaptive scaled additive (ASA) approaches.

| Image type | $\alpha = 0.01$ | | $\alpha = 0.1$ | | $\alpha = 0.2$ | | $\alpha = 0.3$ | |
|---|---|---|---|---|---|---|---|---|
| | SA | ASA | SA | ASA | SA | ASA | SA | ASA |
| Watermarked | 61.84 | 46.41 | 38.83 | 26.56 | 30.82 | 20.68 | 25.16 | 17.35 |
| Noised after watermarked | 20.70 | 20.66 | 20.66 | 19.74 | 20.48 | 17.86 | 19.91 | 16.06 |
| Watermarked & Compressed | 49.32 | 44.56 | 38.60 | 26.54 | 31.07 | 20.70 | 26.03 | 17.49 |

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# Adaptive Scaled Aditive approach- a compromise

**Table:** Peak Signal to Noise Ratio of various watermarked versions of `test_077` image from BSD400 dataset under scaled additive (SA), adaptive scaled additive (ASA) and perceptual forensic (PF) approaches [4].

| Image type | $\alpha = 0.01$ | | |
|---|---|---|---|
| | SA | ASA | PF |
| Watermarked | 61.84 | 46.41 | 75.17 |
| Noised after watermarked | 20.70 | 20.66 | 20.68 |
| Watermarked & Compressed | 49.32 | 44.56 | 38.87 |

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# Visual Comparison

**(a)** Original Image (test_077).

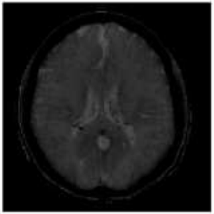
**(b)** Chandra's method output [1].

**(c)** Adaptive method output.

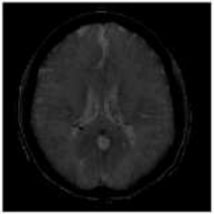
**(d)** Perceptive Method Output with $k = 5$.

**(e)** Perceptive Method with GN ($k = 5$).

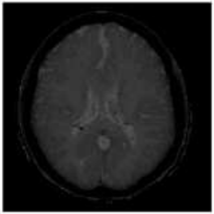
**(f)** Chandra's method with GN.



**(a)** Original Brain CT Image from radiopedia.



**(b)** Scaled Additive Watermarked Image



**(c)** Perceptual Forensic Watermarked Image

**Figure:** Comparison of Brain CT images: (a) Original Brain CT Image, (b) Watermarked with scaled additive approach, (c) Watermarked with perceptual forensic approach.

# Experimental Analysis - Quality Metrics

- **Metrics**:
  - **MSE**: Mean squared pixel difference.
  - **PSNR**: Higher values indicate better signal quality.
  - **SSIM**: Structural similarity closer to 1 represents better quality.
- **Optimization**:
  - Dynamic Thresholding: Balances noise reduction and detail preservation.
  - Adaptive Truncation: Tailors $k$ to desired PSNR/SSIM levels.

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# Challenges and Future Directions

- **Challenges**:
  - High computational load for large images.
  - Optimizing truncation across applications.
- **Future Directions**:
  - Hybrid SVD and alternative denoising methods.
  - Expanding SVD applications to real-time digital forensics.

# Conclusion

- **SVD as a versatile tool**: Effective in image compression, denoising, and watermarking.
- **Key Outcomes**:
  - Reliable quality retention across image applications.
  - Noise reduction with high structural fidelity.
- **Future Potential**: Explore adaptive, hybrid SVD methods for advanced applications.

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# Thank you

Thank you very much for your patient listening